Wykład.

Przykłady z rozkładu mikrokanonicznego:

1) Paramagnetyki.

Przykład 4.3 (cd. przykładów 3.2 i 3.6).

Cel. Prawo Curie-Weisse: $M \propto \frac{B}{T}$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{B,N=\text{const.}}$$

Dane:

1) Liczba mikrostanów: $\boxed{W = \binom{N}{N_+}}$

2) Liczba spinów skierowanych "do góry" (tj. o wartości $s_i = +1$)

$N_+ = ?$ $\quad M = \sum_{i=1}^{N} \mu_i \Rightarrow$ i $\mu_i = \mu_0 s_i$

$$M = \mu_0 \sum_{i=1}^{N} s_i$$

$$M = \mu_0 (N_+ - N_-) \Rightarrow \cdot \quad N = N_+ + N_-$$
$$N_- = N - N_+$$

$$M = \mu_0 (2N_+ - N)$$

$$\boxed{N_+ = \frac{N}{2} + \frac{M}{2\mu_0}}$$

3) Energie pojedynczego momentu magnet: ε_i oraz całości E

$$\boxed{\varepsilon_i = -\mu_i B = -\mu_0 s_i B}$$

$$E = \sum_{i=1}^{N} \varepsilon_i$$

$$E = -\mu_0 B \sum_{i=1}^{N} s_i$$

$$\boxed{E = -\cancel{\mu_0} B \cdot M}$$

3) Wyniki ogólne:

$$\frac{1}{T}=\left(\frac{\partial S}{\partial E}\right)_{B,N} \quad ; \quad \boxed{S=k_B \ln W}$$

podst. relacja termod.
$dE=TdS-pdV+\mu dN$
w gazach

w magnet.
$p\equiv B$ – pole magnet.
$V\equiv M$ – magnet.
wtaj w ... (?)
$N=const$
$\boxed{dE=TdS-BdM}$

$$\frac{1}{T}=\left(\frac{\partial S}{\partial N_+}\frac{\partial N_+}{\partial M}\frac{\partial M}{\partial E}\right)_{BN}$$

$$\boxed{\frac{1}{T}=\left(\frac{\partial S}{\partial N_+}\right)_{B,N}\cdot\left(\frac{\partial M}{\partial N_+}\right)^{-1}\cdot\left(\frac{\partial E}{\partial M}\right)^{-1}}$$

liczymy kolejne pochodne cząstkowe:

$$\frac{\partial S}{\partial N_+}=\left|\; S=k_B\ln\left(\frac{N!}{N_+!(N-N_+)!}\right)\right|=$$

$$=\frac{\partial}{\partial N_+}\left[k_B\left(\ln N!-\ln(N_+!)-\ln((N-N_+)!)\right)\right]=$$

$$=\left\{\begin{array}{l}\ln(N!)\overset{N\gg 1}{\simeq} N\ln N-N\\ \text{wzór Stirlinga}\end{array}\right\}=\cancel{k_B\frac{\partial}{\partial N}\left[-N_++N_+\right.}$$

$$=k_B\frac{\partial}{\partial N_+}\left[-N_+\ln N_+ +\cancel{N_+}-(N-N_+)\ln(N-N_+)+\cancel{(N-N_+)}+N\ln N-\cancel{N}\right]$$

$$=k_B\frac{\partial}{\partial N_+}\left[-N_+\ln N_+-(N-N_+)\ln(N-N_+)+N\ln N\right]$$

$$=k_B\left[-\cancel{\frac{N_+}{N_+}}-\ln N_++\ln(N-N_+)+\cancel{(N-N_+)\cdot\frac{1}{(N-N_+)}}\right]$$

$$=\boxed{k_B\left[\ln\frac{N_-}{N_+}\right]=\frac{\partial S}{\partial N_+}}$$

$$\frac{\partial N_+}{\partial M} = \frac{1}{2\mu_0}$$

(3)

$$\left(\frac{\partial E}{\partial M}\right)^{-1} = \left(-B\right)^{-1} = -\frac{1}{B}$$

stąd:

$$\frac{1}{T} = k_B \ln\left(\frac{N_-}{N_+}\right)\left(-\frac{1}{B}\right)\cdot\left(\frac{1}{2\mu_0}\right)$$

$$\ln\left(\frac{N_+}{N_-}\right) = \frac{2B\mu_0}{k_B T} = \frac{2\mu_0}{k_B}\,\frac{B}{T}$$

$$\frac{N_+}{N-N_+} = e^{\frac{2\mu_0 B}{k_B T}}$$

$$N_+ = \left(N-N_+\right) e^{\frac{2\mu_0 B}{k_B T}}$$

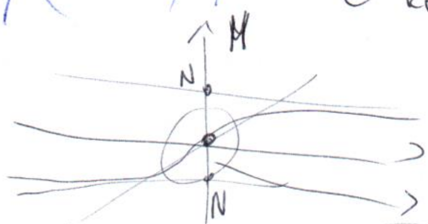
$$N_+\left(1+e^{\frac{2\mu_0 B}{k_B T}}\right) = N e^{\frac{2\mu_0 B}{k_B T}}$$

$$\boxed{N_+ = N\,\frac{e^{\frac{2\mu_0 B}{k_B T}}}{1+e^{\frac{2\mu_0 B}{k_B T}}}}$$

$$N_- = N - N_+ = N\left(1 - \frac{e^{\frac{2\mu_0 B}{k_B T}}}{1+e^{\frac{2\mu_0 B}{k_B T}}}\right) = \boxed{N\,\frac{1}{1+e^{\frac{2\mu_0 B}{k_B T}}} = N_-}$$

4) czyli

$$M = \mu_0\left(N_+ - N_-\right) = \mu_0 N \left(\frac{e^{\frac{2\mu_0 B}{k_B T}} - 1}{e^{\frac{2\mu_0 B}{k_B T}} + 1}\right) = \mu_0 N\left(\frac{e^{\frac{\mu_0 B}{k_B T}} - e^{-\frac{\mu_0 B}{k_B T}}}{e^{\frac{\mu_0 B}{k_B T}} + e^{-\frac{\mu_0 B}{k_B T}}}\right) = \mu_0 N \,\mathrm{tgh}\left(\frac{\mu_0 B}{k_B T}\right)$$



> liniowy obszar, gdy $x \simeq 0$ tj.

$$\boxed{M \propto N\,\frac{\mu_0 B}{k_B T} \sim \frac{B}{T}} \longleftarrow \frac{\mu_0 B}{k_B T} \simeq 0 \text{ mamy } \mathrm{tgh}\,x \simeq x$$