

Significant figures and rounding off results in scientific notation

Whenever you make a measurement, the number of meaningful digits that you write down implies the error in the measurement. For example if you say that the length of an object is 0.428 m, you imply an uncertainty of about 0.001 m. To record this measurement as either 0.4 m or 0.42819667 m would imply that you only know it to 0.1 m in the first case or to 0.00000001 m in the second. You should only report as many significant figures as are consistent with the estimated error. The quantity 0.428 m is said to have three significant figures, that is, three digits that make sense in terms of the measurement. Notice that this has nothing to do with the "number of decimal places". The same measurement in centimeters would be 42.8 cm and still be a three significant figure number.

Students frequently are confused about when to count a zero as a significant figure. The rule is: If the zero has a non-zero digit anywhere to its left, then the zero is significant, otherwise it is not. For example 5.00 has 3 significant figures; the number 0.0005 has only one significant figure, and 1.0005 has 5 significant figures. A number like 300 is not well defined. Rather one should write 3×10^2 , one significant figure, or 3.00×10^2 , 3 significant figures.

For experimental data the error in a quantity defines how many figures in the final result are significant.

The accepted convention for rounding off results is that the uncertainty should be rounded off to one or two significant figures. If the leading figure in the uncertainty is a 1, we use two significant figures, otherwise we use one significant figure. Then the result should be rounded to match.

Example: Round off $z = 12.0349$ cm, where $\Delta z = 0.153$ cm.

Since Δz begins with a 1, we round off Δz to two significant figures: $\Delta z = 0.15$ cm. Hence, round z to have the same number of decimal places: $z = (12.03 \pm 0.15)$ cm.

When the answer is given in scientific notation, the uncertainty should be given in scientific notation with the same power of ten. Thus, if

$$z = 1.43 \times 10^6 \text{ s and } \Delta z = 2 \times 10^4 \text{ s}$$

we should write our result as

$$z = (1.43 \pm 0.02) \times 10^6 \text{ s}$$

This notation makes the range of values most easily understood. The following is technically correct, but is hard to understand at a glance.

$$z = (1.43 \times 10^6 \pm 2 \times 10^4) \text{ s} \quad \text{Don't write like this!}$$

Further examples: Express the following results in proper rounded form, $x \pm \Delta x$.

(a) $m = 14.34506 \times 10^{-3}$ kg, $\Delta m = 0.04251$ grams

(b) $t = 0.02346$ sec, $\Delta t = 1.623 \times 10^{-3}$ sec

(c) $M = 7.35 \times 10^{22}$ kg, $\Delta M = 2.6 \times 10^{20}$ kg

(d) $m = 9.11 \times 10^{-33}$ kg, $\Delta m = 2.2345 \times 10^{-33}$ kg

(e) $a = 23.043$ m/s², $\Delta a = 4.321$ m/s²

(f) $V = 0.067$ m³, $\Delta V = 1.053$ dm³

Answer:

(a) $m = (14.35 \pm 0.04) \text{ g} = (14.35 \pm 0.04) \times 10^{-3} \text{ kg}$

(b) $t = (23.5 \pm 1.6) \times 10^{-3} \text{ s}$

(c) $M = (7.35 \pm 0.03) \times 10^{22} \text{ kg}$

(d) $m = (9 \pm 2) \times 10^{-33} \text{ kg}$

(e) $a = (23 \pm 4) \text{ m/s}^2$

(f) $V = (67.0 \pm 1.1) \text{ dm}^3 = (67.0 \pm 1.1) \times 10^3 \text{ m}^3$